Mesonic Anapole Form Factors of the Nucleons

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Abstract

The chiral quark model posits pseudoscalar and vector meson couplings to constituent quarks. The parity violating meson-quark couplings lead to anapole moments and form factors of the nucleons. These arise both as parity violating meson loop fluctuations as well as exchange currents and polarization currents that are induced by the parity violating interaction between quarks. Because of cancellations between the different contributions the magnitude of the calculated anapole moments is only of the order $\sim 10^{-8}$ and is determined mainly by magnitude of the parity-violating meson-nucleon coupling constants.

1 Introduction

Experimental determination of the strangeness form factors of the nucleons aims at the coupling of the Z^0 boson to the strange quarks and measures the interference term between the γ and Z^0 exchange interactions between electrons and protons in electron-nucleons scattering [1, 2, 3]. If the nucleons have non-zero anapole moments, these contribute an axial current component to the electron-nucleon coupling, which has to be taken into account as a radiative correction in the extraction of the strangeness form factors from the measured asymmetry [4].

Anapole moments are induced by parity violating meson fluctuations of the nucleons, as e.g. $W^{\pm}$ fluctuations [5] and π-loops with one parity violating (PV) vertex [6]. For the nucleons the anapole form factors may be defined as $F_A^{p,n}(0) = F_A^S(q^2) \pm F_A^V(q^2)$, where $F_A^S(q^2)$ and $F_A^V(q^2)$ are the isoscalar and isovector form factors in the current matrix element

$$< p'|j_\mu^A(0)|p >= ie\bar{u}(p')\frac{F_A^S(q^2) + \tau_3 F_A^V(q^2)}{m_N^2}[q^2\gamma_\mu\gamma_5 - 2im_N q_\mu\gamma_5]u(p). \tag{1.1}$$

Here m_N is the nucleon mass, and q is the momentum transfer to the nucleon ($q = p' - p$). The space favoring metric $q^2 = \vec{q}^2 - q_0^2$ will be employed throughout.

The anapole moment due to pionic fluctuations of the proton was estimated in ref. [6] to be $-0.12 f_\pi$, where f_π is the PV pion-nucleon coupling constant, the standard value of which is $-4.5 \cdot 10^{-7}$ [7, 8]. An anapole moment this small does not affect the empirical extraction of the strangeness magnetic moment of the proton in PV electron scattering. A significant effect from the anapole moment would require that it be larger by at least an order of magnitude.

A calculation of the anapole moment of the proton based on the chiral quark model is reported here. As the pseudoscalar and vector mesons couple directly to constituent quarks in this model, the anapole moment arises as a sum of contributions from PV meson loop fluctuations of the constituent quarks and PV meson exchange currents along with PV "polarization currents" induced by the PV meson exchange interaction between quarks. Current conservation demands the presence of all of these combined. The calculation is therefore reminiscent of the calculation of nuclear anapole moments in refs. [5, 6]. This provides an alternate approach to the baryonic loop calculations, which recently have been recast into the form of chiral perturbation theory [9, 10]. The calculation based on the chiral quark model allows a unified treatment of baryon structure and the baryon spectrum based on the same Hamiltonian. It takes all baryonic intermediate states into account, as the constituent quarks lack excited states. A major difference between the quark model and effective baryon field theory approaches is that while the $SU(6)$ aspect of the baryon wave function plays plays a major role in the former it plays none whatever in the latter. The magnetic moments of the baryons provide an example of a set of observables for which the 3-quark structure of the baryon wave function plays the leading role, and meson loops give but small contributions in the quark model.

In the chiral quark model the magnitude of the calculated anapole moment is determined by the PV meson-nucleon coupling constants. By taking into account both the pion and vector meson exchange contributions, the value for the anapole moment of the proton is found to be $-0.9 \cdot 10^{-8}$, with a very large uncertainty margin, that mostly is set by the poorly known values of the PV meson-nucleon coupling constants. The pion contributions alone give rise to a positive value for the anapole moment ($+1.88 \cdot 10^{-8}$), in agreement with results that are obtained in chiral perturbation theory [9]. With the "recommended" values for the PV meson-nucleon coupling constants [7] there are strong cancellations between the pion and vector meson contributions to the anapole moment. The pionic contribution to the anapole moment is found to be somewhat smaller in the chiral quark model than in the pion loop calculation at the baryon level [6] because of a tendency to cancellation between the pion loop and pion exchange current contributions.

This paper falls into 7 sections. In sections 2 and 3 the pion loop and pion exchange and polarization current contributions respectively are calculated. In sections 4 and 5 the corresponding vector meson loop contributions are derived. In section 6 the $\rho\pi$ and $\omega\pi$ loop contributions are calculated. Section 7 contains a discussions of the results and their implications.

2 Pion loop contributions to the anapole form factors

The parity violating (PV) and parity conserving (PC) couplings to constituent quarks have the expressions:

$$\mathcal{L}_{\pi qq}^{PV} = g_{\pi qq}^{W} \bar{\psi}(\vec{\tau} \times \vec{\phi})_3 \psi, \tag{2.1a}$$

$$\mathcal{L}_{\pi qq}^{PC} = i\frac{f_{\pi qq}}{m_\pi} \bar{\psi}\gamma_5\gamma_\mu\partial_\mu\vec{\phi} \cdot \vec{\tau}\psi. \tag{2.1b}$$

Here ψ represents the field of the constituent u and d quarks and $\vec{\phi}$ the pion field. By means of standard quark model algebra the pion-quark coupling constants may be expressed in terms of the corresponding pion-nucleon coupling constants as

$$g_{\pi qq}^{W} = g_{\pi NN}^{W} = -\frac{f_\pi}{\sqrt{2}}, \tag{2.2a}$$

$$f_{\pi qq}^{PC} = \frac{3}{5} f_{\pi NN} \simeq 0.6. \tag{2.2b}$$

Here f_π is the PV πN coupling in the notation of ref. [11] and $f_{\pi NN}$ is the pseudovector πN-coupling.

The PV pion fluctuations of the constituent quarks, which contribute to the anapole moments of the constituent quarks are illustrated by the Feynman diagrams in Fig. 1. For the evaluation of these loop amplitudes the pion and quark current operators are e

$$j_\mu^\pi = e(\partial_\mu \vec{\phi} \times \vec{\phi})_3, \tag{2.3a}$$

$$j_\mu^q = ie\bar{\psi}(\frac{1}{6} + \frac{\tau_3}{2})\gamma_\mu \psi. \tag{2.3b}$$

Here it has been assumed that the anomalous Pauli terms in the e.m. current of the constituent quarks are unimportant [12, 13].

Minimal substitution of the e.m. field $\vec{A}$ in the chiral coupling (2.1b) generates a contact coupling term:

$$\mathcal{L}_{\pi\gamma qq} = ie\frac{f_{\pi qq}}{m_\pi}\bar{\psi}\gamma_5\gamma_\mu A_\mu(\vec{\phi} \times \vec{\tau})_3\psi, \tag{2.4}$$

which generates contact coupling diagrams in addition to the pion loop diagrams in Fig. 1. For e.m. couplings, the amplitudes that are obtained with the PV coupling (2.1b) and the contact coupling (2.4) are equivalent to those obtained with the pseudoscalar coupling

$$\mathcal{L}_{\pi qq} = ig_{\pi qq}\bar{\psi}\gamma_5\vec{\phi} \cdot \vec{\tau}\psi, \tag{2.5}$$

if $g_{\pi qq} = (2m_q/m_\pi)f_{\pi qq}$ (m is the constituent quark mass).

The contributions from the pion loop fluctuations in Figs. 1 to the e.m. current of the $u[d]$ quarks may be expressed as

$$\begin{aligned} j_\mu = &-[+]2eg_{\pi qq}^W g_{\pi qq}\int \frac{d^4p}{(2\pi)^4}\frac{(k_b+k_a)_\mu}{(k_b^2+m_\pi^2)(k_a^2+m_\pi^2)}\{\frac{1}{\gamma\cdot p - im}\gamma_5 - \gamma_5\frac{1}{\gamma\cdot p - im}\} \\ &+2e\frac{1}{3}[-\frac{2}{3}]g_{\pi qq}^W g_{\pi qq}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m_\pi^2}\{\frac{1}{\gamma\cdot p_b - im} + \gamma_\mu\frac{1}{\gamma\cdot p_a - im}\gamma_5 \\ &-\gamma_5\frac{1}{\gamma\cdot p_0 - im}\gamma_\mu\frac{1}{\gamma\cdot p - im}\}. \end{aligned} \tag{2.6}$$

Here $k_b = p_{out} - p$, $k_a = p_{in} - p$ in the first integral (Fig. 1a) and $p_b = p_{out} - k$, $p_a = p_{in} - k$ in the second integral (Fig. 1b). The overall coefficients are those for u-quarks, whereas the overall coefficients for the d-quarks are given in the square brackets [...]. The amplitudes (2.6) contain the appropriate flavor factors (Table 1).

The current operator (2.6) satisfies the current conservation constant $q_\mu j_\mu = 0$ only by addition of the contributions to the current from the self-energy diagrams in Fig. 2. These divergent terms do not contribute to the q-dependence of the form factor. The calculation of the anapole moment therefore should be carried out by dropping these terms and enforcing current conservation on the loop amplitudes (2.6) after subtraction of the divergent unphysical pole terms. The current transversality requirement may be imposed directly by replacement of the operators γ_μ and $(k_a + k_b)_\mu$ in (2.6) by

$$\gamma_\mu \to \gamma_\mu - \frac{\gamma \cdot q}{q^2} q_\mu \tag{2.7a}$$

$$k_{a_\mu} + k_{b_\mu} \to k_{a_\mu} + k_{b_\mu} - \frac{q \cdot (k_a + k_b)}{q^2} q_\mu. \tag{2.7b}$$

This procedure is equivalent to projecting out the longitudinal component from the final result.

Pions are assumed to decouple from constituent quarks at the chiral symmetry restoration scale $\Lambda_\chi \sim 4\pi f_\pi \sim 1.2$ GeV. The loop integrals should accordingly be cut off at or about that momentum scale. This should be done so as to maintain the transversality condition $q_\mu j_\mu = 0$. If in the quark coupling (second) term in (2.5) the opinion propagator is replaced by

$$\frac{1}{m_\pi^2 + k^2} \to v(k^2) = \frac{1}{m_\pi^2 + k^2} (\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + k^2})^2, \tag{2.8}$$

the transversality condition is maintained, provided that the product of the pion propagators in the pion coupling (first) term in (2.5) is replaced by [14]:

$$\frac{1}{k_b^2 + m_\pi^2} \frac{1}{k_a^2 + m_\pi^2} \to \frac{v(k_b^2) - v(k_a^2)}{k_a^2 - k_b^2}. \tag{2.9}$$

Once the anapole moments F_A^u and F_A^d of the $u-$ and d-quarks respectively have been calculated from the expression (2.6), the corresponding proton- and neutron anapole form factors are obtained by the standard quark model expressions as

$$\frac{F_A^p}{m_N^2} = \frac{4}{3} \frac{F_A^u}{m^2} - \frac{1}{3} \frac{F_A^d}{m^2}, \tag{2.10a}$$

$$\frac{F_A^n}{m_N^2} = \frac{4}{3} \frac{F_A^d}{m^2} - \frac{1}{3} \frac{F_A^u}{m^2}. \tag{2.10b}$$

The expressions for the anapole form factors of the proton and the neutron are then found to be

$$F_A^p(q^2) = \frac{m_N^2}{m^2}\{F_\pi(q^2) + \frac{2}{3}F_q(q^2)\}, \tag{2.11a}$$

$$F_A^n(q^2) = \frac{m_N^2}{m^2}\{F_\pi(q^2) + \frac{7}{3}F_q(q^2). \tag{2.11b}$$

Here F_π and F_q are the contributions to the anapole moment of the u-quark from the pion and quark coupling loops (Figs. 1 a,b) in (2.6):

$$F_\pi(q^2) = \frac{m^2}{q^2}\{\frac{g^W_{\pi qq} g_{\pi qq}}{4\pi^2}\int_0^1 dx x \int_0^1 dy\{(log\frac{H_2(\Lambda_\chi^2)}{H_2(m_\pi^2)} - x\frac{\Lambda_\chi^2 - m_\pi^2}{H_2(\Lambda_\chi^2)})$$

$$-(...)_{q^2=0}\}, \tag{2.12a}$$

$$F_q(q^2) = -\frac{m^2}{q^2}\frac{g^W_{\pi qq} g_{\pi qq}}{12\pi^2}\int_0^1 dx(1-x)\int_0^1 dy$$

$$\{([m^2(1-x^2) - q^2(1-x)^2 y(1-y)]K_1(q^2) + log\frac{H_1(\Lambda_\chi^2)}{H_1(m_\pi^2)} - x\frac{\Lambda_\chi^2 - m_\pi^2}{H_1(\Lambda_\chi^2)})$$

$$-(...)_{q^2=0}\}. \tag{2.12b}$$

Here the notation $-(...)_{q^2=0}$ indicates that the value of the preceeding bracket at $q^2 = 0$ should be subtracted from it. This subtraction takes into account the self energy terms.

The auxiliary functions in (2.12) are defined as [13, 15]

$$H_1(M^2) = M^2 x + m^2(1-x)^2 + q^2(1-x)^2 y(1-y), \tag{2.13a}$$

$$H_2(M^2) = M^2 x + m^2(1-x) + q^2 x^2 y(1-y). \tag{2.13b}$$

Finally the function $K_1(q^2)$ is defined as

$$K_1(q^2) = \frac{1}{H_1(m_\pi^2)} - \frac{1}{H_1(\Lambda_\chi^2)} - x\frac{\Lambda_\chi^2 - m_\pi^2}{H_1^2(\Lambda_\chi^2)}. \tag{2.14}$$

The calculated pion loop contributions to the anapole form factors of the proton and the neutron are shown in Fig. 3 as functions of momentum transfer. In the calculation the "standard" value for f_π (2.2a) $-4.5 \cdot 10^{-7}$ [7] was used. The constituent quark mass was set to $m = 340$ MeV and the value of the cut-off Λ_χ was taken to be 1.2 GeV.

The numerical values for pion loop contributions to the anapole moments of the proton and the neutron were found to be $F_A^p(0) = 3.22 \cdot 10^{-8}$ and $F_A^n(0) = 1.45 \cdot 10^{-8}$. These values should be added to those obtained from the pion exchange and polarization currents calculated below. Note similarity between the calculated proton and neutron anapole moments, which indicates that main component of the pion loop contribution to the anapole moment is the isoscalar term, in agreement with other findings [6, 9]. The expression for the pion loop contribution to the isoscalar combination of the anapole form factors here is formally similar to that, which appears in the derivation of the loop contribution at the baryon level with only nucleon intermediate states [6], the only difference being the replacement of quark masses and coupling constants by the corresponding nucleon masses and coupling constants. The expressions for the isovector amplitudes is different however, which is natural, as the quark level calculation takes into account all baryonic intermediate states, and in particular the Δ_{33} resonance.

3 Pion exchange and polarization currents

a. Parity violating pion exchange currents

The pion exchange current operator, which is generated by the pion-quark couplings (2.1), (2.4) and the e.m. currents of the pions and constituent quarks (2.3a) are illustrated diagrammatically in Fig. 4. In the absence of vertex form factors the expressions for the parity violating contact and pion current exchange current operators are

$$j_\mu^\pi(C) = \frac{-ieg_{\pi qq}^W g_{\pi qq}}{2m}[\vec{\tau}^1 \cdot \vec{\tau}^2 - \tau_3^1\tau_3^2]\{\frac{\gamma_5^1\gamma_\mu^1}{k_2^2+m_\pi^2} + \frac{\gamma_5^2\gamma_\mu^2}{k_1^2+m_\pi^2}\}, \tag{3.1a}$$

$$j_\mu^\pi(\pi) = -eg_{\pi qq}^W g_{\pi qq}[\vec{\tau}^1 \cdot \vec{\tau}^2 - \tau_3^1\tau_3^2]\frac{(\gamma_5^2-\gamma_5^1)(k_1-k_2)_\mu}{(k_1^2+m_\pi^2)(k_2^2+m_\pi^2)}. \tag{3.1b}$$

Here the Dirac equation has been invoked for the quarks. The 4-momentum fractions imparted to the two quarks are denoted k_1, k_2 respectively, so that $q = k_1 + k_2$.

The current conservation condition on the PV pion exchange current $j_\mu^{ex}(C) + j_\mu^{ex}(\pi)$ is [16]:

$$\begin{aligned} q_\mu j_\mu^{ex} = &[V_\pi^{PV}(p_1', p_2', p_1+q,\, p_2)j_0^1(p_1+q, p_1) \\ &-j_0^1(p_1', p_1-q)V_\pi^{PV}(p_1'-q, p_2', p_1, p_2)] \\ &+(1\leftrightarrow 2). \end{aligned} \tag{3.2}$$

Here j_0^1 is the charge density operator of a single quark and V_π^{PV} is the parity violating pion exchange potential for constituent quarks. The initial and final 4-momenta of the quark pair are denoted p_1, p_2 and p_1', p_2' respectively. The expression for the PV π-exchange interaction is

$$V_\pi^{PV} = -i g_{\pi qq} g_{\pi qq}^W (\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{\gamma_5^1 - \gamma_5^2}{k^2 + m_\pi^2}. \tag{3.3}$$

A practical consequence of the continuity equation (3.2) is the requirement that the exchange current and exchange induced polarization current contributions combine to the proper non-singular form (1.1).

The pion exchange Yukawa functions in the exchange current operators (3.1) as well as in the potential (3.3) should be cut-off at the chiral restoration scale as were the pion loops. Current conservation is maintained if in the contact current operator (3.1a) and the potential the Yukawa function $1/(k^2 + m_\pi^2)$ is modified as in (2.8), and the product of pion Yukawa functions in the pionic current (3.1b) is replaced as in (2.9) [14].

The current conservation condition (3.2) ensures that the exchange current operator compensates for the non-conservation of the single quark e.m. current in the nucleon states, in the presence of a parity violating pion exchange interaction between the constituent quarks. If the negative parity component of the proton wave function is neglected, transversality has to be enforced on the net exchange current contribution in the calculation of the anapole moment. This procedure will be implemented here, although the polarization current that is induced by the negative parity component is also considered explicitly below.

For the calculation of the matrix elements of the pion exchange current operator it is conducive to rewrite it in the spin representation. To lowest order in the inverse quark masses the operators (3.1) combine to the expression

$$\vec{j}^\pi = -e \frac{g_{\pi qq}^W g_{\pi qq}}{2m} (\vec{\tau}^1 \cdot \vec{\tau}^2 - \tau_3^1 \tau_3^2) \{ \frac{\vec{\sigma}^2}{k_1^2 + m_\pi^2} + \frac{\vec{\sigma}^1}{k_2^2 + m_\pi^2}$$
$$- \frac{(\vec{\sigma}^1 \cdot \vec{k}_1 - \vec{\sigma}^2 \cdot \vec{k}_2)(\vec{k}_1 - \vec{k}_2)}{(k_1^2 + m_\pi^2)(k_2^2 + m_\pi^2)} \}. \tag{3.4}$$

In addition to the parameters that appear in the calculation of the pion loop contributions to the anapole moment the exchange current contribution also depends on the proton wave function and thus on the confinement scale. The orbital part of proton wave function will here be described by the oscillator function (Table 2)

$$\psi(\vec{r}, \vec{\rho}) = (\frac{\omega}{\sqrt{\pi}})^3 e^{-(r^2 + \rho^2)\omega^2/2}, \tag{3.5}$$

which appears translationally invariant quark models as well as in the covariant integrable quark model for the baryons developed in ref. [17, 18]. Here $\vec{r}$ and $\vec{\rho}$ are Jacobi coordinates for the 3-quark system. With a constant flavor-spin hyperfine interaction the baryon spectrum is well described with the value $\omega = 311$ MeV [18], whereas in recent a meson exchange model for the spectrum $\omega = 1240$ MeV [19]. The latter value is most consistent with the present meson exchange model for the anapole moment.

The orbital matrix element of the pion exchange contact current (3.1a) may be written as

$$j_\mu(C) = -\frac{eg^W_{\pi qq}g_{\pi qq}}{2m}\frac{m_\pi}{4\pi}(\vec{\tau}^1 \cdot \vec{\tau}^2 - \tau^1_3\tau^2_3)[i\gamma^1_5\gamma^1_\mu + i\gamma^2_5\gamma^2_\mu]M_\rho(q)M_r(q), \tag{3.6}$$

where $M_\rho(q)$ and $M_r(q)$ are the orbital matrix elements:

$$M_\rho(q) = 4\pi(\frac{\omega}{\sqrt{\pi}})^3\int_0^\infty d\rho\rho^2 j_0(\frac{q\rho}{\sqrt{6}})e^{-\rho^2\omega^2}, \tag{3.7}$$

$$M_r(q) = 4\pi(\frac{\omega}{\sqrt{\pi}})^3\int_0^\infty drr^2 j_0(\frac{qr}{\sqrt{2}})y_0(m_\pi r\sqrt{2})e^{-r^2\omega^2}. \tag{3.8}$$

In the latter integral $y_0(x)$ is a cut-off Yukawa function defined as

$$y_0(m_\pi r\sqrt{2}) = \frac{e^{-m_\pi r\sqrt{2}}}{m_\pi r\sqrt{2}} - (\frac{\Lambda}{m_\pi})\frac{e^{-\Lambda_\chi r\sqrt{2}}}{\Lambda_\chi r\sqrt{2}} - \frac{\Lambda^2_\chi - m^2_\pi}{2\Lambda_\chi m_\pi}e^{-\Lambda_\chi r\sqrt{2}}. \tag{3.9}$$

For proton and neutron states with spin z-component $+1/2$, the matrix element of the spin flavor operator calculated with the wave functions listed in Table 3 is

$$< (\vec{\tau}^1 \cdot \vec{\tau}^2 - \tau^1_3\tau^2_3)(i\gamma^1_5\gamma^2_3 + i\gamma^2_5\gamma^2_3) > \simeq < (\vec{\tau}^1 \cdot \vec{\tau}^2 - \tau^1_3\tau^2_3)(\sigma^1_3 + \sigma^2_3) > = \frac{4}{3}. \tag{3.10}$$

The contribution of pion exchange contact current to the anapole moment of the proton and the neutron then takes the form

$$F^p_{A,C}(q^2) = F^n_{A,C}(q^2) = \frac{2}{3}\frac{m^2_N}{q^2}\frac{g^W_{\pi qq}g_{\pi qq}}{4\pi}\frac{m_\pi}{m}[M_\rho(q)M_r(q) - M_\rho(0)M_r(0)]. \tag{3.11}$$

The subtraction of the matrix element at $q = 0$ is required, unless a consistent calculation of the polarization current is performed, in which case the matrix element at $q = 0$ is cancelled by a corresponding term in the latter. This point will be treated in detail below.

The calculation of the matrix element of the pionic exchange current operator (3.1b) is somewhat more complicated. The result for this contribution to the anapole moment may be cast into the form

$$F^p_{A,\pi}(q^2) = F^n_{A,\pi}(q^2) = -\frac{m_N^2}{q^2}\frac{g^W_{\pi qq}g_{\pi qq}}{\pi}\frac{1}{18m}M_\rho(q)\int_0^1 dx\{(m^*_\pi(x)$$

$$4\pi(\frac{\omega}{\sqrt{\pi}})^3\int_0^\infty drr^2e^{-r^2\omega^2}\{j_0(\xi)[2y_0(m^*_\pi r\sqrt{2})-y_1(m^*_\pi r\sqrt{2})]$$

$$-j_2(\xi)[y_0(m^*_\pi r\sqrt{2})+y_1(m^*_\pi r\sqrt{2})]\}$$

$$-(...)_{q^2=0}\}. \tag{3.12}$$

Here the variable ξ is defined as

$$\xi = qr(\frac{1}{2}-x)\sqrt{2}, \tag{3.13}$$

and the function $y_1(m^*_\pi r\sqrt{2})$ is defined as:

$$y_1(m^*_\pi r\sqrt{2}) = e^{-m^*_\pi r\sqrt{2}} - (\frac{\Lambda^*}{m^*_\pi})e^{-\Lambda^* r\sqrt{2}}$$

$$+\frac{1}{2}\frac{\Lambda^{*2}-m^{*2}_\pi}{m^*_\pi\Lambda^*}(1-\Lambda^* r\sqrt{2})e^{-\Lambda^* r\sqrt{2}}. \tag{3.14}$$

The mass parameters m^*_π and Λ^* are functions of the integration variable x:

$$m^*_\pi(x) = \sqrt{m_\pi^2+q^2x(1-x)}, \tag{3.15a}$$

$$\Lambda^*(x) = \sqrt{\Lambda^2+q^2x(1-x)}. \tag{3.15b}$$

The calculated pion exchange current contributions to the anapole form factors of the proton and the neutron are shown in Fig. 5 The exchange current contributions are shown both as calculated with the oscillator parameters $\omega = 311$ MeV and 1240 MeV in order to obtain an estimate of the theoretical uncertainty of the calculated values. The calculated value of the pion exchange current contribution to the anapole moment of the proton (and the neutron) is -1.80 $\cdot 10^{-8}$ with $\omega = 311$ MeV and $-1.15\cdot 10^{-8}$ with ω =1240 MeV. As pointed out above the larger oscillator parameter is consistent with the meson exchange model for the baryon wave functions in ref. [19, 20]. While the wave function dependence of the anapole moment contribution itself is thus not very strong the exchange current contribution to the anapole form factor is very strong.

For both parameter values a tendency for cancellation between the exchange current and the pion loop contributions to the anapole moment is visible. This then suggests that the net values of the anapole moments of the nucleons will be small and only of the order $\sim 10^{-8}$. In the present quark model calculation the exchange current contribution to the anapole moment is purely isoscalar, but the pion loop contribution is about 2 times larger in the case of the proton than in the case of the neutron. This result differs from that found in ref. [6], where the PV pion loop fluctuations of the proton were considered. The isoscalar nature in that loop calculation is a consequence of dominance of the contributions of the terms, in which the e.m. coupling is to the intermediate pions over the contributions from the terms in which the e.m. coupling is to the intermediate nucleon.

b. Parity violating pion exchange induced polarization current

The parity violating pion exchange interaction (3.3) induces a negative parity component into the proton wave function. In view of the small magnitude of the PV pion exchange potential, this component may be calculated by means of first order perturbation theory as

$$\psi^- = -\sum_n \frac{< n|V_\pi^{PV}|0 >}{E_n - E_0}\psi_n^-, \tag{3.16}$$

where ψ_n^- are the wave functions that describe the negative parity excitations of the nucleons, and $E_n - E_0$ are the corresponding resonance excitation energies.

The electromagnetic current operator of the constituent quarks:

$$\vec{j}_q = e(\frac{1}{6} + \frac{\tau_3}{2})\{\frac{\vec{p}\,' + \vec{p}}{2m} + \frac{i}{2m}\vec{\sigma} \times \vec{q}\}, \tag{3.17}$$

will have non-vanishing matrix elements between the positive (ψ^+) and negative (ψ^-) parity components of the nucleon wave function, which contribute to the anapole form factor of the nucleon. These contributions are referred to as polarization currents [5]. The polarization current then takes the form

$$\begin{aligned}\vec{j}^{pol} = &-\sum_n(\psi^+, \sum_q \vec{j}_q \frac{< n|V_\pi^{PV}|0 >}{E_n - E_0}\,\psi_n^-)\\ &-\sum_n(\psi_n^-\,\frac{< 0|V_\pi^{PV}|n >}{E_n - E_0}, \sum_q \vec{j}_q \quad \psi^+).\end{aligned} \tag{3.18}$$

This expression is illustrated schematically in Fig. 7.

The only $\frac{1}{2}^-$ nucleon states in the P-shell of the baryons are the $N(1535)$ and $N(1650)$ resonances. The former is a member of an $S = 1/2$ negative parity doublet and the latter a member of an $S = \frac{3}{2}$ negative parity quartet, in the usual assignment [21]. With this assignment only the $N(1535)$ resonance contributes to the sum (3.18). As the P-shell lies ~ 600 MeV above the nucleon, it is justified to truncate the sum in (3.18) after the P-shell states, in view of the much larger energy denominators of the resonances in the F and higher shells.

The required matrix element of the PV pion exchange interaction may be evaluated using the oscillator wave functions of the covariant quark model in ref. [18], which are listed in Table 2. The only non-vanishing matrix elements are

$$< N(1535)^+, \frac{1}{2}|V_\pi^{PV}|N(939)^+, \frac{1}{2} >= \mp i\frac{\sqrt{2}}{3}\frac{g_{\pi qq}g_{\pi qq}^W}{4\pi}\frac{m_\pi^2}{m}\mathcal{M}, \tag{3.19}$$

where $\mathcal{M}$ is the radial matrix element

$$\mathcal{M} = \sqrt{2}\omega(\frac{\omega}{\sqrt{\pi}})^3 4\pi \int_0^\infty drr^3 \bar{y}_1(m_\pi r\sqrt{2})e^{-r^2\omega^2}. \tag{3.20}$$

In (3.19) the $-$ sign applies for protons, and the $+$ sign for neutrons. The Yukawa function $\bar{y}_1$ is defined as

$$\bar{y}_1(m_\pi r\sqrt{2}) = (1 + 1/m_\pi r\sqrt{2})\frac{e^{-m_\pi r\sqrt{2}}}{m_\pi r\sqrt{2}} - (\frac{\Lambda_\chi}{m_\pi})^2(1 + 1/\Lambda_\chi r\sqrt{2})\frac{e^{-\Lambda_\chi r\sqrt{2}}}{\Lambda_\chi r\sqrt{2}}$$
$$-\frac{1}{2}((\frac{\Lambda_\chi}{m_\pi}^2 - 1)e^{-\Lambda_\chi r\sqrt{2}}. \tag{3.21}$$

Evaluation of the matrix element (3.18) of the convection current part (3.17) of the single quark current operator yields the following contribution to the anapole form factors of the proton and the neutron:

$$F_{A,conv}^p = F_{A,conv}^n = -\eta\frac{m_N^2}{q^2}\frac{1}{3}\frac{g_{\pi qq}g_{\pi qq}^W}{4\pi}\mathcal{M}\frac{m_\pi^2}{m^2}\frac{\omega}{\Delta}(\frac{\omega}{\sqrt{\pi}})^3 4\pi\int_0^\infty d\rho\rho^2 e^{-\rho^2\omega^2}[j_0(\sqrt{\frac{2}{3}}q\rho) - 1]. \tag{3.22}$$

Here Δ is the energy dominator $\Delta = 1535 - 939 = 596$ MeV. Note that this is an isoscalar term, as required by current conservation with the isoscalar pion exchange current operator considered above. The factor η is a correction factor that has to be chosen so that the continuity equation that links the exchange current and polarization current corrections is satisfied. If all negative parity states in the sum over n in (3.18) are taken into account, this factor would be unity. In order that no spurious pole at $q^2 = 0$ occur in the sum of pion exchange current and polarization current matrix elements appear in the present perturbative calculation of the latter, the value for the correction factor η has to be chosen as

$$\eta = \frac{2}{3}\frac{m\Delta}{\omega^2}. \tag{3.23}$$

If the oscillator parameter ω is taken to be $\omega = 311$ MeV [18], the numerical value for η is 1.4, which indicates that truncating the sum over n in (3.18) is fairly good approximation. In the case of the larger value 1240 MeV [19] for ω the expression (3.23) gives $\eta = 0.09$, which indicates that higher lying excited states contribute significantly.

Once the correction factor η is included in the expression (3.22), there is in principle no need to subtract the values of the matrix elements at $q = 0$ from the exchange (3.11), (3.12) and polarization current contributions (3.22) to the anapole moment. Above the value of the matrix elements at $q = 0$ have been subtracted explicitly, as it gives an explicit indication that the pole term at $q^2 = 0$ has to cancel out.

The spin current component of the single quark current operator (3.17) gives rise to the following contribution to the anapole moment of the proton and the neutron:

$$F_{A,spin}^{p,n} = \eta(1, -\frac{1}{3})\frac{2}{27}\frac{m_N^2}{m^2}\frac{m_\pi^2\mathcal{M}}{\Delta}\frac{g_{\pi qq}g_{\pi qq}^W}{4\pi}\omega(\frac{\omega}{\sqrt{\pi}})^3 4\pi\int_0^\infty d\rho\rho^4 e^{-\rho^2\omega^2}[j_0(\sqrt{\frac{2}{3}}q\rho) + j_2(\sqrt{\frac{2}{3}}q\rho)]. \tag{3.24}$$

Here the term 1 in the first bracket on the r.h.s. applies in the case of the proton and the term $-1/3$ in the case of the neutron.

The pion exchange induced polarization current contributions to the anapole form factors of the neutron and the proton have been plotted along with the corresponding pion exchange current contributions in Figs. 5 and 6. The polarization current contribution to the anapole moment of the nucleon depends strongly on the wave function model. With the wave function parameter value $\omega = 311$ MeV the polarization current contribution to the anapole moment of the proton is $-3.38 \cdot 10^{-8}$ and with the parameter value $\omega = 1240$ MeV it is $-0.19 \cdot 10^{-8}$. The corresponding contributions to the anapole moment of the neutron are $-0.38 \cdot 10^{-8}$ and $-0.02 \cdot 10^{-8}$ respectively.

The net combination of pion exchange and polarization current contributions to the anapole form factors of the proton and the neutron are shown in Fig. 8. The net calculated pionic contribution to the anapole moment of the proton is – including the pion loop contribution - is $1.88 \cdot 10^{-8}$ and that to the anapole moment of the neutron is $0.28 \cdot 10^{-8}$ (for ω = 1240 MeV).

4 Vector meson loop contributions to the anapole moment

4.1 ρ-meson loop contributions

The parity violating ρ-nucleon coupling is by convention written in terms of 3 different flavor coupling terms [11]

$$\mathcal{L} = i\bar{\psi}_N\{h^0_\rho \vec{\tau}\cdot\vec{\rho}_\mu + h^1_\rho \rho^3_\mu + h^2_\rho \frac{3\tau_3\rho^3_\mu - \vec{\tau}\cdot\vec{\rho}_\mu}{2\sqrt{6}}\}\gamma_\mu\gamma_5\psi_N. \tag{4.1}$$

Here ψ_N is the nucleon and $\vec{\rho}_\mu$ the isovector field of the ρ-meson. The coupling constants h_ρ have been determined phenomenologically only within wide uncertainly ranges, the "recommended" values being $h^0_\rho = -30g_w$, $h^1_\rho = -0.5g_w$ and $h^2_\rho = -25g_w$ ($g_w = 3.8\cdot 10^{-8}$) [7]. Standard quark model algebra then implies that the PV coupling of ρ-meson to constituent quarks be

$$\mathcal{L} = i\bar{\psi}\{h^0_{\rho qq}\vec{\tau}\cdot\vec{\rho}_\mu + h^1_{\rho qq}\rho^3_\mu + h^2_{\rho qq}\frac{3\tau_3\rho^3_\mu - \vec{\tau}\cdot\vec{\rho}_\mu}{2\sqrt{6}}\}\gamma_\mu\gamma_5\psi, \tag{4.2}$$

where ψ represents the quark fields. The PV ρ-quark coupling constants $h_{\rho qq}$ are determined by the corresponding ρ-nucleon coupling constants as

$$h^0_{\rho qq} = \frac{3}{5}h^0_\rho,\ h^1_{\rho qq} = h^1_\rho,\ h^2_{\rho qq} = \frac{3}{5}h^2_\rho. \tag{4.3}$$

The PV ρ-meson loop contributions to anapole moments of the constituent u and d quarks are illustrated diagrammatically in Fig. 9. The flavor factors for the different flavor coupling terms h_ρ are listed in Table 1 for the different diagrams. The calculation of these loop contributions require the ρ-quark coupling Lagrangian

$$\mathcal{L}_{\rho qq} = ig_{\rho qq}\bar{\psi}\gamma_\mu\vec{\tau}\cdot\vec{\rho}_\mu\psi, \tag{4.4}$$

where $g_{\rho qq} = g_{\rho NN} \simeq 2.6$ [23] and the e.m. current operator for the ρ-meson

$$j_\mu = \pm ie\{\rho^\dagger_\nu\partial_\mu\rho_\nu - \rho^\dagger_\nu\partial_\nu\rho_\mu\}. \tag{4.5}$$

With these couplings and the flavor factors listed in Table 1 the expressions for the ρ-meson loop amplitudes, for u quarks, where e.m. coupling is to the internal quark (Fig. 9a), takes the form

$$j_\mu(u) = -\frac{1}{3}(2h^1_{\rho qq} + \frac{h^2_{\rho qq}}{\sqrt{6}})e\, g_{\rho qq}\int\frac{d^4k}{(2\pi)^4}\frac{\delta_{\alpha\beta} + \frac{k_\alpha k_\beta}{m_p^2}}{k^2 + m_p^2}$$

$$\{\gamma_\alpha\gamma_5\frac{1}{\gamma\cdot p_b-im}\gamma_\mu\frac{1}{\gamma\cdot p_a-im}\gamma_\beta\gamma_5$$
$$+\gamma_\alpha\frac{1}{\gamma\cdot p_b-im}\gamma_\mu\frac{1}{\gamma\cdot p_a-im}\gamma_\beta\gamma_5\}. \tag{4.6}$$

Here the notation is the same as in eqn. (2.6). The same expression applies for d-quarks provided that the flavor factor $(2h^1_{\rho qq}+h^2_{\rho qq}/\sqrt{6})/3$ with the PV ρ-quark couplings is replaced by the corresponding factor $-(h^0_{\rho qq}+h^1_{\rho qq}/3-h^2_{\rho qq}/\sqrt{6})$.

The expression for the contribution of the corresponding ρ-meson loop to fluctuations to the u-quark current, illustrated in Fig. 9b, where the e.m. coupling is to the ρ-meson is

$$j_\mu(u)=2(h^0_{\rho qq}-\frac{1}{\sqrt{6}}h^2_{\rho qq})eg_{\rho qq}\int\frac{d^4p}{(2\pi)^4}\frac{1}{k_b^2+m_p^2}\frac{1}{k_a^2+m_p^2}$$
$$\{(k_a+k_b)_\mu(\delta_{\alpha\beta}+\frac{k_{a\alpha}k_{a\beta}}{m_\rho^2})(\delta_{\beta\delta}+\frac{k_{b\beta}k_{b\delta}}{m_\rho^2})$$
$$\{\gamma_\delta\gamma_5\frac{1}{\gamma\cdot p-im}\gamma_\alpha+\gamma_\delta\frac{1}{\gamma\cdot p-im}\gamma_\alpha\gamma_5\}$$
$$-\gamma_\delta\gamma_5(\delta_{\beta\delta}+\frac{k_{b\delta}k_{b\beta}}{m_\rho^2})\frac{k_{a\beta}}{\gamma\cdot p-im}(\delta_{\mu\alpha}+\frac{k_{a\mu}k_{a\alpha}}{m_\rho^2})\gamma_\alpha$$
$$-\gamma_\delta(\delta_{\beta\delta}+\frac{k_{b\delta}k_{b\beta}}{m_\rho^2})\frac{k_{a\beta}}{\gamma\cdot p-im}(\delta_{\mu\alpha}+\frac{k_{a\mu}k_{a\alpha}}{m_\rho^2})\gamma_a\gamma_5$$
$$-\gamma_\alpha\gamma_5(\delta_{\mu\alpha}+\frac{k_{b\mu}k_{b\alpha}}{m_\rho^2})\frac{k_{b\beta}}{\gamma\cdot p-im}(\delta_{\beta\delta}+\frac{k_{a\delta}k_{a\beta}}{m_\rho^2})\gamma_\delta$$
$$-\gamma_\alpha(\delta_{\mu\alpha}+\frac{k_{b\mu}k_{b\alpha}}{m_\rho^2})\frac{k_{b\beta}}{\gamma\cdot p-im}(\delta_\beta\delta+\frac{k_{a\delta}k_{a\beta}}{m_\rho^2})\gamma_\delta\gamma_5\}. \tag{4.7}$$

The corresponding operator for ρ-meson fluctuations of the d-quark in this case is obtained simply by change of the overall sign. The sum of the current operators (4.6) and (4.7) satisfy the tranversality condition when added to the combination of ρ-meson self energy diagrams (cf. Fig. 2). As those only add a q-independent infinite contribution, which has to be subracted, they are dropped here out and the transversality condition on the ρ-meson loop currents (4.1) and (4.7) is enforced as in the case of the pion loop diagrams above.

The current operators (4.6) and (4.7) simplify significantly by leaving out the terms that are inversely proportional to m_ρ^2 in the numerator of the ρ-meson propagators. As those terms have been found to be of very small numerical significance for $q^2<m_p^2$ in other

calculations of related type [15, 24], they are dropped here, especially as their effect is small in comparison to that of the wide uncertainty in the PV ρ-meson coupling constants [7].

The contributions to the anapole moments of the u and d quarks from the ρ-meson loops with the e.m. coupling to the internal quark line (Fig. 9 a) and to the ρ-meson (Fig. 9 b), respectively, are then

$$F_{A,q}^{u,d}(q^2) = f^{u,d}\frac{m^2}{q^2}\frac{g_{\rho qq}}{4\pi^2}\int_0^1 dx(1-x)\int_0^1 dy$$

$$\{[m^2(1-x)^2 - q^2x - q^2(1-x)^2y(1-y)]\bar{K}_1(q^2, m_\rho^2)$$

$$+log\frac{H_1(\Lambda_\chi^2)}{H_1(m_\rho^2)} - x\frac{\Lambda_\chi^2 - m_\rho^2}{H_1^2(m_\rho^2)}\} - \{...\}_{q^2=0}, \qquad (4.8a)$$

$$F_{A,\rho}^{u,d}(q^2) = g^{u,d}\frac{m^2}{q^2}\frac{g_{\rho qq}}{4\pi^2}\int_0^1 dxx\int_0^1 dy$$

$$\{[2m^2x(1-x) + xq^2 + 2x^2q^2y(1-y)]\bar{K}_2(q^2, m_\rho^2)$$

$$+6log\frac{H_2(\Lambda_\chi^2)}{H_2(m_\rho^2)} - 6x\frac{\Lambda_\chi^2 - m_\rho^2}{H_2^2(m_\rho^2)}\} - \{...\}_{q^2=0}. \qquad (4.8b)$$

Here $f^{u,d}$ and $g^{u,d}$ are the flavor factors

$$f^u = \frac{2h_{\rho qq}^1}{3} + \frac{h_{\rho qq}^2}{\sqrt{6}}, \quad f^d = h_{\rho qq}^0 + \frac{h_{\rho qq}^1}{3} - \frac{h_{\rho qq}^2}{\sqrt{6}}, \qquad (4.9a)$$

$$g^u = h_{\rho qq}^0 - \frac{h_{\rho qq}^2}{2\sqrt{6}}, \quad g^d = -h_{\rho qq}^0 + \frac{h_{\rho qq}^2}{2\sqrt{6}}. \qquad (4.9b)$$

The functions $K_1(q^2)$ and $K_2(q^2)$ are defined as (cf. ref. [15])

$$\bar{K}_1(q^2, m_\rho^2) = \frac{1}{H_1(m_\rho^2)} - \frac{1}{H_1(\Lambda_\chi^2)} - x\frac{\Lambda_\chi^2 - m_\rho^2}{H_1^2(\Lambda_\chi^2)}, \qquad (4.10a)$$

$$\bar{K}_2(q^2, m_\rho^2) = \frac{1}{H_2(m_\pi^2)} - \frac{1}{H_2(\Lambda_\chi^2)} - x\frac{\Lambda_\chi^2 - m_\rho^2}{H_2^2(\Lambda_\chi^2)}. \qquad (4.10b)$$

The ρ–meson loop contributions to the anapole moments of the proton and the neutron may then be calculated using eqs. (2.10). In Fig. 10 the calculated anapole form factors of the proton and the neutron are shown. The magnitudes are the similar to those given by the corresponding pion loop contributions, but the signs are opposite. In the numerical calculations the "recommended" values [7] for the PV ρ–nucleon coupling constants were employed. The calculated values of the ρ–meson loop contributions to the anapole moments of the proton and the neutron were found to be $-0.44 \cdot 10^{-8}$ and $1.52 \cdot 10^{-8}$ respectively.

The $\rho-$meson loops here are those, which are induced by the PV $\rho-$meson-constituent quark couplings, which in turn are implied by the $\rho-$nucleon couplings (4.1). In addition to these, there may also appear a PV $\rho\rho\gamma$ coupling, akin to the anapole moment term in the e.m. current operator. That coupling has been considered in ref. [5] and would generate further PV loop contributions to the constituent quark current. These are conserved by the form of the coupling, and inversely proportional to m_ρ^2. In view of the uncertain value of the PV $\rho\rho\gamma$ coupling strength and as the terms of order m_ρ^{-2} were dropped from the loop calculation above, we have for consistency not included these loop contributions in the present calculation.

4.2 ω-meson loop contributions

The standard expression for the parity violating ω-nucleon coupling is

$$\mathcal{L} = i\bar{\psi}_N\{h_\omega^0\omega_\mu + h_\omega^1\tau^3\omega_\mu\}\gamma_\mu\gamma_5\psi_N. \tag{4.11}$$

The "recommended values" for the PV ω-nucleon coupling constants are $h_\omega^0 = -5g_w$ and $h_\omega^1 = -3g_w$, with a large uncertainty margin [7, 11].

The corresponding PV coupling of ω mesons to constituent quarks is

$$\mathcal{L} = i\bar{\psi}\{h_{\omega qq}^0\omega_\mu + h_{\omega qq}^1\tau^3\omega_\mu\}\gamma_\mu\gamma_5\psi. \tag{4.12}$$

The $SU(6)$ quark model for the nucleon wave functions leads to the following relations between the PV ω-nucleon and the corresponding PV ω-quark coupling constants:

$$h_{\omega qq}^0 = \frac{h_\omega^0}{3}, \quad h_{\omega qq}^1 = \frac{3}{5}h_\omega^1. \tag{4.13}$$

We shall rely on these relations here.

For the calculation of the contributions from the PV ω-meson loop fluctuations of the constituent u and d quarks to their anapole moments, the ω-quark coupling

$$\mathcal{L} = ig_{\omega qq}\bar{\psi}\gamma_\mu\omega_\mu\psi \tag{4.14}$$

is employed. The ω-quark coupling constant determined from the corresponding ω-nucleon vector coupling constant $g_{\omega NN}$ is $g_{\omega qq} = g_{\omega NN}/3$ by the quark model. The recent Nijmegen model [25] for the nucleon-nucleon interaction determines $g_{\omega NN} = 10.35$, from which it follows that $g_{\omega qq} = 3.45$. This value is used here.

Because of the neutrality of the ω meson the e.m. field only couples to the internal quark in the PV ω meson loop contributions. The expressions for these contributions to the anapole moments of the u and d quarks, then take the form

$$F_{A,\omega}^{u,d}(q^2) = -\frac{m^2}{q^2}(h_{\omega qq}^0 \mp h_{\omega qq}^1)\int_0^1 dx(1-x)\int_0^1 dy$$
$$\{[m^2(1-x^2) - q^2 x - q^2(1-x)^2 y(1-y)]\bar{K}_1(q^2, m_\omega^2)$$
$$+log\frac{H_1(\Lambda_\chi^2)}{H_1(m_\omega^2)} - x\frac{\Lambda_\chi^2 - m_\rho^2}{H_1^2(m_\rho)}\} - \{...\}_{q^2=0}. \tag{4.15}$$

The expressions for the ω-loop contributions to the nucleon anapole form factors are then given by the equations (2.10). In Fig. 10 the numerical values for the ω-loop contributions to the anapole form factors of the proton and the neutron are also shown. The numerical values for the ω-loop contributions to the anapole moments of the nucleons are found to be $-0.097\cdot 10^{-8}$ (proton) and $0.34\cdot 10^{-8}$ (neutron) (Table 4). The ω meson exchange current contributions of to the anapole moments are expected to be very small in view of the neutrality of the $\omega-$meson and the small values of the PV ω coupling constants as compares to the corresponding $\rho-$meson couplings..

5 Vector meson exchange and polarization currents

a. Parity violating ρ meson exchange currents

The PV ρ-meson exchange current operators, which are implied by the ρ-quark couplings (4.12), (4.4) are illustrated diagrammatically in Fig. 11. The contact current is customarily derived as a pair current operator [5, 16], but may also be derived as a contact current as in the case of the pion exchange current $j_\mu(C)$ (3.1a) above. To see this one may rewrite the the parity conserving ρ-quark coupling (4.4) as

$$\mathcal{L}_{\rho qq} = g_{\rho qq}\bar{\psi}((p'+p)_\mu - \frac{1}{2m}\sigma_{\mu\nu}\partial_\nu)\vec{\rho}_\mu \cdot \vec{\tau}\psi. \tag{5.1}$$

Minimal substitution of the e.m. vector potential A_μ in this coupling yields the contact coupling

$$\mathcal{L}_{\gamma\rho qq} = e\frac{g_{\rho qq}}{2m}\bar{\psi}\{\sigma_{\mu\nu}A_\nu(\vec{\rho}_\mu \times \vec{\tau})_3 - A_\mu(\vec{\rho}_\mu \cdot \vec{\tau} + \rho_{\mu 3}\}\psi \tag{5.2}$$

in analogy with (2.4) in the case of the pion.

The expressions for the ρ-meson contact and ρ-current exchange current operators are found to be

$$j_\mu^\rho(C) = -ie\frac{g_{\rho qq}}{2m}\frac{1}{k_2^2+m_\rho^2}\{(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{2\sqrt{6}})(\vec{\tau}^1\times\vec{\tau}^2)_3\gamma_\nu^2\gamma_5^2\sigma^1_{\nu\mu}$$

$$[(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{2\sqrt{6}})\vec{\tau}^1\cdot\vec{\tau}^2 + (h^0_{\rho qq} - \frac{h^2_{\rho qq}}{\sqrt{6}})\tau_3^2 + h^1_{\rho qq}(1+\tau_3^1) + \frac{3}{2\sqrt{6}}\tau_3^1\tau_3^2)]\} + (1\leftrightarrow 2), \quad (5.3a)$$

$$j_\mu^\rho(\rho) = ieg_{\rho qq}(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{2\sqrt{6}})\frac{(\vec{\tau}^1\times\vec{\tau}^2)_3}{(k_1^2+m_\rho^2)(k_2^2+m_\rho^2)}$$

$$\{(k_1-k_2)_\mu[\gamma_\nu^2\gamma_5^2\gamma_\nu^1 + \gamma_\nu^1\gamma_5^1\gamma_\nu^2]$$

$$-\gamma_\nu^2\gamma_5^2k_{1\nu}\gamma_\mu^1 + \gamma_\nu^1\gamma_5^1k_{2\nu}\gamma_\mu^2 + \gamma_\mu^2\gamma_5^2\gamma_\nu^1k_{2\nu} - \gamma_\mu^1\gamma_5^1\gamma_\nu^2k_{1\nu}\}. \quad (5.3b)$$

In the spin representation the sum of the local parts of the ρ-meson exchange current operators (5.3), to lowest order in $1/m^2$, reduce to the expression

$$\vec{j}^\rho = e(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{2\sqrt{6}})\frac{g_{\rho qq}}{2m}(\vec{\tau}^1\times\vec{\tau}^2)_3$$

$$\{(\vec{\sigma}^1\times\vec{\sigma}^2)\{\frac{1}{k_1^2+m_\rho^2} + \frac{1}{k_2^2+m_\rho^2}\} - \frac{1}{(k_1^2+m_\rho^2)(k_2^2+m_\rho^2)}$$

$$\{(\vec{\sigma}^1\times\vec{\sigma}^2)\cdot(\vec{k}_1-\vec{k}_2)(\vec{k}_1-\vec{k}_2) + \vec{\sigma}^2\cdot\vec{k}_1\vec{\sigma}^1\times\vec{k}_1 - \sigma^1\cdot k_2\vec{\sigma}^2\times\vec{k}_2 - \vec{\sigma}^2\vec{\sigma}^1\cdot\vec{k}_1\times\vec{k}_2 + \vec{\sigma}^1\vec{\sigma}^2\cdot\vec{k}_1\times\vec{k}_2\}\}$$

$$+e\frac{g_{\rho qq}}{2m}\{\frac{\vec{\sigma}^2}{k_2^2+m_\rho^2}[(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{2\sqrt{6}})\vec{\tau}^1\cdot\vec{\tau}^2 + (h^0_{\rho qq} - \frac{h^2_{\rho qq}}{\sqrt{6}})\tau_3^2 + h^1_{\rho qq}(1+\tau_3^1) + \frac{3}{2\sqrt{6}}\tau_3^1\tau_3^2)]$$

$$+\frac{\vec{\sigma}^1}{k_1^2+m_\rho^2}[(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{2\sqrt{6}})\vec{\tau}^1\cdot\vec{\tau}^2 + (h^0_{\rho qq} - \frac{h^2_{\rho qq}}{\sqrt{6}})\tau_3^1 + h^1_{\rho qq}(1+\tau_3^2) + \frac{3}{2\sqrt{6}}\tau_3^1\tau_3^2)]. \quad (5.4)$$

Apart from the coupling constants, this PV ρ-meson exchange current operator has a formal similarity to that of the corresponding PV pion exchange current operator (3.4). In contrast to the latter the ρ-meson exchange current operator is an isovector operator.

Comparison of the expressions (3.4) and (5.4) for the PV π and ρ meson exchange current operator makes it possible to derive the ρ-meson exchange contribution to the anapole moments of the proton and the neutron by analogy. The required spin-isospin matrix elements are given in Table 3.

The contribution of the seagull current (5.3a) to the anapole moment of the proton and the neutron becomes becomes

$$F_{A,C}(q) = 4\frac{m_N^2}{q^2}h_C(p,n)\frac{g_{\rho qq}}{4\pi}$$
$$\frac{m_\rho}{m}[M_\rho(q)\bar{M}_r(q) - M_\rho(0)\bar{M}_r(0)]. \tag{5.5}$$

Here the coefficients $h_C(p,n)$ are defined as

$$h_C(p,n) = \pm(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{2\sqrt{6}}) - \frac{1}{2}(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{2\sqrt{6}}) \mp \frac{5}{12}(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{\sqrt{6}}) \mp \frac{h^1_{\rho qq}}{4} + \frac{1}{12}h^1_{\rho qq} - \frac{h^2_{\rho qq}}{8\sqrt{6}}. \tag{5.6}$$

The upper and lower signs in this expression applies to protons and neutrons respectively. The function $M_\rho(0)$ is defined in (3.7), and the matrix element $\bar{M}_r(q)$ is defined as

$$\bar{M}_r(q) = 4\pi(\frac{\omega}{\sqrt{\pi}})^3\int_0^\infty drr^2 j_0(\frac{qr}{\sqrt{2}})y_0(m_\rho\sqrt{2}r)e^{-r^2\omega^2} \tag{5.7}$$

The cut-off Yukawa function y_0 is defined in (3.9), the ρ-meson mass here having been substituted for the pion mass.

The contribution to the anapole moment from the PV ρ-meson exchange current operator (5.3b) (second term in (5.4)), where the e.m. coupling is to the ρ-meson field, takes the form (cf. (3.12))

$$F^p_{A,\rho}(q) = -F^n_{A,n}(q) = -\frac{m_N^2}{q^2}(h^0_{\rho qq} - \frac{h^2_{\rho qq}}{2\sqrt{6}})\frac{g_{\rho qq}}{4\pi}\frac{1}{m}M_\rho(q)$$
$$\int_0^1 dx\{m^*_\rho(x)4\pi(\frac{\omega}{\sqrt{\pi}})^3\int_0^\infty drr^2e^{-r^2\omega^2}$$
$$\{2j_0(\xi)[2y_0(m^*_\rho r\sqrt{2}) - (1-\frac{\vec{q}^{\,2}}{8m^{*2}_\rho})y_1(m^*_\rho r\sqrt{2})]$$
$$-j_2(\xi)[y_0(m^*_\rho r\sqrt{2}) + y_1(m^*_\rho r\sqrt{2})] - (...)_{q=0}\}. \tag{5.8}$$

Here the notation is the same as used in the expression (3.11). The mass function $m^*_\rho(x)$ is defined as (cf. 3.15a)

$$m^*_\rho(x) = \sqrt{m^2_\rho + q^2x(1-x)}. \tag{5.9}$$

The net calculated ρ-meson exchange current contributions $F^\rho_A(q,C) + F^\rho_A(q,\rho)$ to the anapole form factors of the proton and the neutron are shown in Figs. 12 and 13 respectively for two different values of the wave function parameter ω. With $\omega = 311$ MeV these contributions to the anapole moments of the proton and the neutron are $-1.18 \cdot 10^{-8}$ and $-0.85 \cdot 10^{-8}$ respectively. With the larger value $\omega = 1240$ MeV, which is consistent with the Hamiltonian model for the baryon spectrum in ref. [19, 20], the corresponding values are $-1.79 \cdot 10^{-8}$ and $0.73 \cdot 10^{-8}$ respectively.

In ref. [5] another type of PV $\rho-$meson exchange current was considered. That exchange current arises from the PV $\rho\rho\gamma$ coupling, which corresponds to the anapole coupling of the γ to the nucleons. This isoscalar exchange current is conserved by construction, and inversely proportional to m_ρ^2. Because of the uncertain value of the PV $\rho\rho\gamma$ coupling strength and as the terms of order m_ρ^{-2} have not been included in the calculation above, this isoscalar $\rho-$meson exchange current has not been included here for reasons of consistency.

b. Parity violating ρ exchange induced polarization current

The parity violating ρ meson exchange interaction between constituent quarks contributes to the negative parity component of the proton wave function. This interaction takes the form [11]

$$\begin{aligned} V_\rho^{PV} &= -\frac{g_{\rho qq}}{4\pi}\{h_{\rho qq}^0 \vec{\tau}^1 \cdot \vec{\tau}^2 + \frac{1}{2} h_{\rho qq}^1 (\tau_3^1 + \tau_3^2) \\ &\quad + \frac{1}{2\sqrt{6}} h_{\rho qq}^2 (3\tau_3^1\tau_3^2 - \vec{\tau}^1 \cdot \vec{\tau}^2)\} \\ &\{(\vec{\sigma}^1 - \vec{\sigma}^2)\cdot\{\frac{\vec{p}_1 - \vec{p}_2}{2m}, f(r)\}_+\} + i\vec{\sigma}^1 \times \vec{\sigma}^2 \cdot [\frac{\vec{p}_1 - \vec{p}_2}{2m}, f(r)]\} \\ &\quad + \frac{g_{\rho qq}}{4\pi}\frac{h_{\rho qq}^1}{2}(\tau_3^1 - \tau_3^2)(\vec{\sigma}^1 + \vec{\sigma}^2)\cdot[\frac{\vec{p}_1 - \vec{p}_2}{2m}, f(r)]. \end{aligned} \tag{5.10}$$

The radial Yukawa function $f(r)$ is defined as

$$f(r) = \frac{e^{-m_\rho r}}{r} - \frac{e^{-\Lambda_\chi r}}{r} - \frac{m_\rho^2}{2\Lambda_\chi}(\frac{\Lambda_\chi^2}{m_\rho^2} - 1)e^{-\Lambda_\chi r}. \tag{5.11}$$

Here $r = |\vec{r}_1 - \vec{r}_2|$. The ρ-exchange induced PV polarization current is then given by the general expression (3.18) with V_ρ^{PV} substituted in place of V_π^{PV}. In order to be consistent with the approximate treatment of the $\rho-$meson exchange current operators above, where only the local part of the operator was retained, the $\rho-$meson exchange induced polarization contribution is calculated here using only the local (commutator) terms in the $\rho-$meson exchange interaction.

If the sum over negative parity states is approximated by the contributions from the lowest lying negative parity resonances, $N(1535)$ and $\Delta(1620)$, the only matrix elements that are required are

$$< N(1535)^+, \frac{1}{2}|V_\rho^{PV}|N(939)^+, \frac{1}{2} >= i\sqrt{2}\frac{g_{\rho qq}}{4\pi}\frac{m_\rho^2}{m}(\frac{h_{\rho qq}^0}{2} \mp \frac{h_{\rho qq}^1}{3})\mathcal{M}_\rho, \tag{5.12a}$$

$$< \Delta(1620)^+, \frac{1}{2}|V_\rho^{PV}|N(939)^+, \frac{1}{2} >= -2i\sqrt{2}\frac{g_{\rho qq}}{4\pi}\frac{m_\rho^2}{m}(\pm\frac{h_{\rho qq}^1}{6} + \frac{h_{\rho qq}^2}{2\sqrt{6}})\mathcal{M}_\rho^a. \tag{5.12b}$$

Here the orbital matrix element $\mathcal{M}_\rho$ is defined as

$$\mathcal{M}_\rho = \sqrt{2}\omega(\frac{\omega}{\sqrt{\pi}})^3 4\pi\int_0^\infty drr^3 e^{-r^2\omega^2}\bar{y}_1(m_\rho r\sqrt{2}), \tag{5.13}$$

Here the wave function models in Table 2 has again been employed. The function $\bar{y}_1$ is defined as in (3.12).

Evaluation of the matrix element (3.18) of the convection current part of the single quark current (3.17), with V_π^{PV} replaced by V_ρ^{PV}, with the sum over n truncated to include only the contributions from the $N(1535)$ and $\Delta(1620)$ negative parity resonances then leads to the following anapole form factor contributions to the proton and the neutron:

$$F_{A,conv}^p = -F_{A,conv}^n = \eta\frac{m_N^2}{q^2}\frac{g_{\rho qq}}{4\pi}\frac{m_\rho^2}{m^2}\{\frac{1}{\Delta_1}(\frac{h_{\rho qq}^0}{2} \mp \frac{1}{3}h_{\rho qq}^1)\mathcal{M}_\rho$$
$$+\frac{2}{\Delta_2}((\mp\frac{h_{\rho qq}^1}{6} + \frac{h_{\rho qq}^2}{2\sqrt{6}})\mathcal{M}_\rho\}\omega(\frac{\omega}{\sqrt{\pi}})^3 4\pi\int_0^\infty d\rho\rho^2 e^{-\rho^2\omega^2}[j_0(\sqrt{\frac{2}{3}}q\rho) - 1]. \tag{5.14}$$

Here Δ_1 and Δ_2 are the mass differences $m[N(1535)] - m_N = 596$ MeV and $m[\Delta(1620) - m_N] = 681$ MeV respectively. The same renormalization correction factor η (3.23) as was employed in the expression for the pion exchange induced parity violating polarization current has been used here. The required renormalization constant was derived explicitly from the continuity equation in the case of pion exchange. It cannot be calculated in the same way in the case of ρ-meson exchange, because the PV ρ-coupling (4.1) does not satisfy the required transversality condition.

The spin current component of the single quark current operator (3.17) gives the following ρ-meson exchange induced polarization current contribution to the anapole form factors of the proton and the neutron:

$$F_{A,spin}^{p,n} = -\eta(1, -\frac{1}{3})\frac{2}{9}\frac{g_{\rho qq}}{4\pi}\frac{m_N^2 m_\rho^2}{m^2}\{\frac{1}{\Delta_1}(\frac{h_{\rho qq}^0}{2} \mp \frac{1}{3}h_{\rho qq}^1)\mathcal{M}_\rho$$

$$+\frac{2}{3\Delta_2}((\pm\frac{h^1_{\rho qq}}{6}+\frac{h^2_{\rho qq}}{2\sqrt{6}})\mathcal{M}_\rho\}$$

$$\omega(\frac{\omega}{\sqrt{\pi}})^3 4\pi\int_0^\infty d\rho\rho^4 e^{-\rho^2\omega^2}[j_0(\sqrt{\frac{2}{3}}q\rho)+j_2(\sqrt{\frac{2}{3}}q\rho)]. \tag{5.15}$$

Here the term 1 in the bracket $(1,-1/3)$ on the r.h.s. applies in the case of the proton and the term $-1/3$ in the case of the neutron.

The $\rho-$meson exchange induced contributions to the anapole form factor of the proton is shown in Fig. 12 as obtained with two different values for the wave function parameter ω. The corresponding contributions to the anapole form factor of the neutron are shown in Fig. 13. For $\omega = 1240$ MeV the ρ-exchange induced polarization current contribution to the anapole moment of the proton is $+0.088\cdot 10^{-8}$ and that of the neutron is $-0.047\cdot 10^{-8}$ (Table 4).

6 Meson loops with transition couplings

In addition to the pion and vector meson loop contributions to the anapole form factors, loop contributions with e.m. pseudoscalar-vector meson transition couplings also contribute to the anapole form factors. These are illustrated by the Feynman diagrams in Fig. 14.

The $\rho\rightarrow\pi\gamma$ transition coupling may be descibed by the current matrix element

$$<\pi^a(k')|j_\mu(0)|\rho^b_\nu(k)>=-i\frac{g_{\rho\pi\gamma}}{m_\rho}\epsilon_{\mu\nu\lambda\sigma}k_\lambda k'_\sigma\delta^{ab}. \tag{6.1}$$

The $\rho\pi\gamma$ coupling constant may be calculated from the known radiative widths of the ρ-mesons: $\Gamma(\rho^\pm\rightarrow\pi^\pm\gamma)=68$ keV, $\Gamma(\rho^0\rightarrow\pi^0\gamma)=119$ keV. These yield $g_{\rho^\pm\pi^\pm\gamma}=0.57$ and $g_{\rho^0\pi^0\gamma}=0.75$.

The corresponding transition current matrix element for the $\omega\rightarrow\pi^0\gamma$ transition is

$$<\pi^0(k')|j_\mu(0)|\omega_\nu(k)>=-i\frac{g_{\omega\pi\gamma}}{m_\omega}\epsilon_{\mu\nu\lambda\sigma}k_\lambda k'_\sigma. \tag{6.2}$$

From the radiative width of the ω-meson: $\Gamma(\omega\rightarrow\pi^0\gamma)=717$ MeV one obtains $g_{\omega\pi\gamma}=1.84$.

The current operators that describe the $\rho\rightarrow\pi\gamma$ transition loops on u- and d-quarks in Fig. 9, where one of the hadronic vertices is parity violating takes the form

$$j^{u,d}_\mu=-i\frac{G^{u,d}}{m_\rho}\int\frac{d^u p}{(2\pi)^4}\{\frac{\epsilon_{\mu\nu\lambda\sigma}k_{a\lambda}k_{b\sigma}}{(k_b^2+m_\pi^2)(k_a^2+m_\rho^2)}\frac{1}{\gamma\cdot p-im}\gamma_\nu$$

$$+\gamma_\nu \frac{1}{\gamma p - im} \frac{\epsilon_{\mu\nu\lambda\sigma} k_{a\lambda} k_{b\sigma}}{(k_b^2 + m_\rho^2)(k_a^2 + m_\pi^2)}\}. \tag{6.3}$$

Here the coupling constant combinations G^u and G^d have been defined as

$$G^{u,d} = 2g^W_{\pi qq} g_{\rho qq} g_{\rho^\pm \pi^\pm \gamma} + g_{\pi qq}[2h^0_{\rho qq} g_{\rho^\pm \pi^\pm \gamma}$$

$$+(h^0_{\rho qq} \pm h^1_{\rho qq} + \frac{h^2_{\rho qq}}{\sqrt{6}}) g_{\rho^0 \pi^0 \gamma}]. \tag{6.4}$$

The only difference between G^u and G^d is in the sign of the $h^1_{\rho qq}$ term, which is positive in the case of G^4 and negative in the case of G^d.

Because of the presence of the Levi-Civita symbol in the current expressions (6.3), the apparent ultraviolet divergences drop out. Similarly the terms $k_\alpha k_\beta / m_\rho^2$ in the vector meson propagators drop out. In line with the cutting off of the loop integrals of the pion and vector meson loop contributions above at the chiral symmetry restoration scale, a similar cutoff is applied here by insertion of a form factor,

$$F(k_\pi^2, k_\rho^2) = \frac{\Lambda_\chi^2 - m_\pi^2}{\Lambda_\chi^2 + k_\pi^2} \frac{\Lambda_\chi^2 - m_\rho^2}{\Lambda_\chi^2 + k_\rho^2}, \tag{6.5}$$

in the integrands, where k_π and k_ρ denote the 4-momenta of the π and ρ mesons.

To reduce expressions (6.3) to standard form (1.1), it is convenient to employ the relation

$$\epsilon_{\mu\nu\lambda\sigma}(p' + p)_\lambda q_\sigma \gamma_\nu = q^2 \gamma_\mu \gamma_5 - 2im\gamma_5 q_\mu, \tag{6.6}$$

where $q_\mu = p'_\mu - p_\mu$. the PV $\rho - \pi$ loop contributions to the anapole moments of the u and d quarks then become

$$F^{u,d}_{A,\rho\pi}(q^2) = \frac{G^{u,d}}{16\pi^2} \frac{m}{m_\rho} \int_0^1 dx x \int_0^1 dy (1-x)[1 + x(1-2y)]$$

$$\{\frac{1}{G(m_\rho, m_\pi)} - \frac{1}{G(m_\rho, \Lambda_\chi)} - \frac{1}{G(\Lambda_\chi, m_\pi)} + \frac{1}{G(\Lambda_\chi, \Lambda_\chi)}\}. \tag{6.6}$$

Here the function $G(m_1, m_2)$ has been defined as

$$G(m_1, m_2) = m^2(1-x)^2 + m_1^2 x(1-y) + m_2'^2 xy + q^2 x^2 y(1-y). \tag{6.7}$$

The contribution from the PV loops that involve the $\omega \to \pi\gamma$ transition in the meson line may be calculated by the same method. This contribution obtains no contribution from the

PV π-quark coupling (2.1a), which only contributes to loop amplitudes with charged pions. The PV $\pi\gamma$ transition loops leads to the following anapole form factors of the u and d quarks:

$$F_{A,\omega\pi}^{u,d}(q^2) = \frac{H^{u,d}}{16\pi^2}\frac{m}{m_\omega}\int_0^1 dxx\int_0^1 dy(1-x)[1+x(1-2y)]$$

$$\{\frac{1}{G(m_\omega,m_\pi)}-\frac{1}{G(m_\omega,\Lambda_\chi)}-\frac{1}{G(\Lambda_\chi,m_\omega)}+\frac{1}{G(\Lambda_\chi,\Lambda_\chi)}\}. \tag{6.8}$$

Here a cut-off factor of the form (6.5) with m_ρ replaced by m_ω has been included in the expression. The factors $H^{u,d}$ represent the coupling constant combinations

$$H^{u,d} = -g_{\pi qq}(h_{\omega qq}^0 \pm h_{\omega qq}^1)g_{\omega\pi\gamma}. \tag{6.9}$$

The $\rho\pi$ and $\omega\pi$ loop contributions to the anapole moment of the proton, as calculated using the expressions (2.10) have been plotted in Fig. 15. The contribution from the $\rho\pi$ loops is the larger one because of the larger PV $\rho-$quark coupling strengths. These contribute $-0.67\cdot 10^{-8}$ and $-0.65\cdot 10^{-8}$ to the anapole moments of the proton and the neutron (Table 4). The corresponding contributions from the $\omega\pi$ loops to the anapole moments of the proton and the neutron are $0.12\cdot 10^{-8}$ and $-0.035\cdot 10^{-8}$ respectively.

The $\rho\to\pi\gamma$ and $\omega\to\pi\gamma$ transition couplings also give rise to parity violating exchange current operators. These are illustrated diagrammatically in Fig. 16. The corresponding 2-quark currents may be derived by using the meson-nucleon couplings in sections 2 and 4 and the $\rho\to\pi\gamma$ and $\omega\to\pi\gamma$ transition couplings (6.1) and (6.2).

These exchange current operators do not however contribute to the anapole form factors of the nucleon. The exchange current operator that involves the PV pion-quark coupling (2.1a) is spin-independent, and vanishes in nucleon states because of the antisymmetric Levi-Civita symbol in the e.m. pion-vector meson transition couplings (6.1) and (6.2). The exchange currents that involve the PV vector meson couplings (4.2) (4.11) have no local component, which would contribute to the anapole form factor.

7 Discussion

The calculated net anapole form factors of the proton and the neutron are shown in Fig. 17. These results take into account all the loop and exchange current contributions calculated above. As a satisfactory description of the nucleon radii with the constituent quark model of ref. [20] requires that a form factor be associated with the constituent quarks [19], the calculated anapole form factors have been multiplied by the corresponding form factor in

Fig. 17. The form of the constituent quark form factor used was $exp(-r^2q^2/6)$, where the mean square radius value is $r^2 = 0.133$ fm^2 [19]. The calculated anapole moments are also shown without this quark form factor modification.

The corresponding contributions to the anapole moments of the proton and the neutron are listed in Table 4. The calculated value of the anapole moment of the proton is $-0.90 \cdot 10^{-8}$, the magnitude of whichwhich is considerably smaller than what is obtained by considering only the pionic contributions. The main difference from the PV pion-nucleon loop estimate in ref. [6] and the chiral perturbation theory estimate in ref. [9] is the appearance of exchange and polarization current contributions at the quark level. The present calculation suggests that the net mesonic anapole moment is far too small to lead to any notable correction to the extraction of the strangeness magnetic moment in the SAMPLE experiment [1, 2]. The net mesonic contribution to the anapole moment of the neutron is $0.68 \cdot 10^{-8}$.

The calculated anapole moment has a wide $\sim 100\%$ theoretical uncertainty, which mainly arises from the large uncertainty in the PV meson-nucleon couplings [7], which are used as input parameters in the calculation. The loop amplitudes have a cut-off sensitivity, but this is fairly small, once the cut-off is taken to be of the order of the chiral symmetry restoration scale. The exchange current and polarization current contributions have an additional dependence on the baryon wave function model, which to some extent is limited by requirement of consistency with the calculated baryon spectrum [20].

A main qualitative result of the present calculation is that at the quark level there is a tendency for cancellation between the loop and exchange current contributions and the tendency for cancellation between pionic and vector meson contributions.

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Table 1

Flavor factors in the loop integrals, when the parity conserving coupling preceeds the parity violating one. The factors with an asterisk change sign when the ordering of the couplings is reversed.

$u \to \pi^+ d \to u$ $u \to \pi^0 u \to u$ $d \to \pi^- u \to d$ $d \to \pi^0 d \to d$	$2i^*$ 0 $2i^*$ 0
$u \to \rho^+ d \to u$	$2 \quad (h^0_{\rho qq})$ $0 \quad (h^1_{\rho qq})$ $-1/\sqrt{6} \quad (h^2_{\rho qq})$
$u \to \rho^0 u \to u$	$1 \quad (h^0_{\rho qq})$ $1 \quad (h^1_{\rho qq})$ $1/\sqrt{6} \quad (h^2_{\rho qq})$
$d \to \rho^- u \to d$	$2 \quad (h^0_{\rho qq})$ $-1/\sqrt{6} \quad (h^2_{\rho qq})$
$d \to \rho^0 d \to d$	$1 \quad (h^0_{\rho qq})$ $-1 \quad (h^1_{\rho qq})$ $1/\sqrt{6} \quad (h^2_{\rho qq})$
$u \to \rho^0 u \to u$	$1(h^0_{\omega qq})$ $1(h^1_{\omega qq})$
$d \to \rho^0 u \to u$	$1(h^0_{\omega qq})$ $-1(h^1_{\omega qq})$

Table 2

Explicit wave functions for the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ non-strange baryon states in the lowest S- and P-shells. The functions φ_{nlm} are harmonic oscillator wave functions. The subscripts $\pm$ on the spin-isospin states denote the Yamanouchi symbols (112) and (121) respectively [17].

p, n	$\frac{1}{\sqrt{2}}\varphi_{000}(\vec{\rho})\varphi_{000}(\vec{r})\{\|\frac{1}{2}, t>_+ \|\frac{1}{2}, s>_+ + \|\frac{1}{2}, t>_- \|\frac{1}{2}s>_-\}$
$N(1535)$	$\frac{1}{2}\sum_{m\sigma}(1, \frac{1}{2}, m, \sigma\|\frac{1}{2}s)\{\varphi_{01m}(\vec{\rho})\varphi_{000}(\vec{r})$ $[\|\frac{1}{2}, t>_+ \|\frac{1}{2}\sigma>_+ - \|\frac{1}{2}, t>_- \|\frac{1}{2}, \sigma>_-]$ $+\varphi_{000}(\vec{\rho})\varphi_{01m}(\vec{r})[\|\frac{1}{2}, t>_+ \|\frac{1}{2}, \sigma>_- + \|\frac{1}{2}, t>_- \|\frac{1}{2}, \sigma>_+]\}$
$\Delta(1620)$	$\frac{1}{\sqrt{2}}\sum_{m\sigma}(1, \frac{1}{2}, m, \sigma\|\frac{1}{2}, s)\{\varphi_{01m}(\vec{\rho})\varphi_{000}(\vec{r})\|\frac{3}{2}, t> \|\frac{1}{2}, \sigma>_+$ $+\varphi_{000}(\vec{\rho})\varphi_{01m}(\vec{r})\|\frac{3}{2}, t> \|\frac{1}{2}, \sigma>_-\}$
$N(1650)$	$\frac{1}{\sqrt{2}}\sum_{m\sigma}(1, \frac{3}{2}, m, \sigma\|\frac{1}{2}, s)\{\varphi_{01m}(\vec{\rho})\varphi_{000}(\vec{r})\|\frac{1}{2}, t>_+$ $+\varphi_{000}(\vec{\rho})\varphi_{01m}(\vec{r})\|\frac{1}{2}, t>_-\}\|\frac{3}{2}, \sigma>$

Table 3

Spin-flavor matrix elements of exchange current operators for protons and neutrons with $s_z = +1/2$.

Operator	$< p, \frac{1}{2}\|0\|p, \frac{1}{2} >$	$< n, \frac{1}{2}\|0\|n, \frac{1}{2} >$
$(\vec{\tau}^1 \cdot \vec{\tau}^2 - \tau_3^1 \tau_3^2)(\sigma_3^1 + \sigma_3^2)$	4/3	4/3
$(\vec{\tau}^1 \times \vec{\tau}^2)_3 (\vec{\sigma}^1 \times \vec{\sigma}^2)_3$	-4	4
$\sigma_3^1 + \sigma_3^2$	2	2
$\sigma_3^1 \tau_3^1 + \sigma_3^2 \tau_3^2$	$\frac{10}{3}$	$-\frac{10}{3}$
$\sigma_3^1 \tau_3^2 + \sigma^2 \tau_3^1$	$-\frac{2}{3}$	$\frac{10}{3}$
$(\sigma_3^1 + \sigma_3^2) \tau^1 \cdot \tau^2$	2	2

Table 4

Mesonic contributions to the anapole moment of the proton and the neutron.

	proton	neutron
π loops	$+3.22 \cdot 10^{-8}$	$+1.45 \cdot 10^{-8}$
π exchange	$-1.15 \cdot 10^{-8}$	$-1.15 \cdot 10^{-8}$
π polarization	$-0.19 \cdot 10^{-8}$	$-0.02 \cdot 10^{-8}$
ρ loops	$-0.44 \cdot 10^{-8}$	$+1.52 \cdot 10^{-8}$
ρ exchange	$-1.79 \cdot 10^{-8}$	$+0.73 \cdot 10^{-8}$
ρ polarization	$+0.088 \cdot 10^{-8}$	$-0.047 \cdot 10^{-8}$
ω loops	$-0.097 \cdot 10^{-8}$	$+0.34 \cdot 10^{-8}$
$\rho\pi$ loops	$-0.67 \cdot 10^{-8}$	$-0.65 \cdot 10^{-8}$
$\omega\pi$ loops	$+0.12 \cdot 10^{-8}$	$-0.035 \cdot 10^{-8}$
Total	$-0.90 \cdot 10^{-8}$	$+0.68 \cdot 10^{-8}$

Figure Captions

Figure 1. Pion loop contributions to the anapole form factors of the constituent quarks. The square vertex represents the parity violating pion-quark coupling (2.1a). To the current operator should be added the contributions with the PV and PC hadronic vertices permuted.

Figure 2. Self energy contributions to the anapole moments of the constituent quarks that are required for current conservation. The square vertex represents the parity violating pion-quark coupling (2.1a). To these should be added the corresponding contributions with the PV and PC hadronic vertices permuted.

Figure 3. Calculated pion loop contributions to the anapole form factor of the proton and the neutron.

Figure 4. Pion exchange current contributions to the anapole form factors of the nucleons. The fermion lines represent constituent quarks. The square vertex represents the PV pion-quark coupling (2.1a). The inclusion of the terms with the PV and PC hadronic vertices should be understood.

Figure 5. Pion exchange current and polarization current contributions to the anapole form factor of the proton. The PV pion exchange current contribution is denoted "PI EXC" and the pion exchange induced polarization current contribution is denoted PI POL. The curves A and B where obtained with the values ω =311 MeV and ω =1240 MeV respectively for the quark wave function parameter ω (3.5)

Figure 6. Pion exchange current and polarization current contributions to the anapole form factor of the neutron. The PV pion exchange current contribution is denoted "PI EXC" and the pion exchange induced polarization current contribution is denoted PI POL. The curves A and B where obtained with the values ω =311 MeV and ω =1240 MeV respectively for the quark wave function parameter ω (3.5)

Figure 7. Schematic representation of the pion polarization current that arises from the PV pion exchange interaction (3.3) induced negative parity admixture in the nucleon wave function. The presence additional terms with the PC and PV pion-quark vertices permuted should be understood.

Figure 8. Combined pion exchange current and polarization current contributions to the anapole form factors of the proton and the neutron.

Figure 9. Vector meson loop contributions to the anapole form factors of constituent quarks. The square vertex represents the parity-violating couplings of vector mesons to constituent quarks ((4.2) and (4.12). These should be combined with the loop amplitudes with the PV and PC hadronic vertices permuted.

Figure 10. $\rho-$ and $\omega-$ meson loop contributions to the anapole form factors of the proton and the neutron.

Figure 11. Parity violating ρ meson exchange current contributions to the anapole form factors of the nucleon. The square vertex represents the PV ρ meson coupling to constituent quarks (4.2). The inclusion of the terms with the PV and PC hadronic vertices should be understood.

Figure 12. $\rho-$meson exchange current (EXC) and polarization current contributions (POL) to the anapole form factor of the proton. The curves A and B where obtained with the values ω =1250 MeV and ω =311 MeV respectively for the quark wave function parameter ω (3.5).

Figure 13. $\rho-$meson exchange current (EXC) and polarization current contributions (POL) to the anapole form factor of the neutron. The curves A and B where obtained with the values ω =1250 MeV and ω =311 MeV respectively for the quark wave function parameter ω (3.5).

Figure 14. Parity violating $\rho\pi$ and $\omega\pi$ loop contributions to the anapole form factors of the nucleons. The square vertex represents the parity violating pion-quark coupling (2.1a).

Figure 15. The $\rho\pi$ and $\omega\pi$ loop contributions to the anapole form factors of the proton and the neutron.

Figure 16. Mixed vector-pseudoscalar exchange current operators.

Figure 17. The net meson loop and exchange current contributions to the anapole form factor of the proton and the neutron. The results calculated without constituent quark form factors are shown separately.